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32. Logic and Philosophy of Logic

**REPRESENTING INTUITIONISM PHILOSOPHY OF LOGIC BY A LOGICALLY FORMALIZED MULTIMODAL AXIOMATIC EPISTEMOLOGY SYSTEM Ф0**

**(Was** **Brouwer** **Quite Satisfied by** **Heyting’s Intuitionistic Logic Formalization?)**

**МОДЕЛИРОВАНИЕ ИНТУИЦИОНИСТСКОЙ ФИЛОСОФИИ ЛОГИКИ ПОСРЕДСТВОМ ЛОГИЧЕСКИ ФОРМАЛИЗОВАННОЙ МУЛЬТИМОДАЛЬНОЙ АКСИОМАТИЧЕСКОЙ СИСТЕМЫ ЭПИСТЕМОЛОГИИ Ф0**

**(Был ли Брауэр вполне удовлетворен формализацией интуиционистской логики Гейтингом?)**

By the middle of 20th century, specialists in philosophy of mathematics became tired of the controversy among the formalists, the intuitionists, and the logicists which had arrived into a deadlock and became not interesting for proper mathematicians. However, even to the present time there is no consistent solution of the antinomy-problem. The well-known compromise has been created by A. Heyting’s intuitionistic logic formalization. But there is a strong feeling that the proper philosophical core of Brouwer’s intuitionism and systematical anti-logicism has been ignored by Heyting intuitionistic logic. Perhaps, the concession underlying the compromise had been not acceptable for L.E.J. Brouwer’s philosophy of mathematics. The present paper submits an alternative for Heyting option of consistent uniting (synthesizing) D. Hilbert’s formalism and Brouwer’s intuitionism (anti-logicism) in philosophy of mathematics. The alternative option is realized by precise formulating syntax and semantics of/for a formal multimodal axiomatic theory Ф0 of universal epistemology uniting (synthesizing) a-priori-ism and empiricism in general and in philosophy of logic and mathematics especially.

Keywords: Brouwer, intuitionism-philosophy-of-logic, Heyting, Hilbert, law-of-logic

Hilbert’s philosophy of logic [1], [2] seems incompatible with Brouwer’s intuitionism attitude to logic laws [3], [4], [5]. In relation to the intuitionistic philosophy of logic and its relations with formalism and logicism, the question in subtitle of this paper is very interesting, but the set of relevant *facts* (not legends conveyed by oral talks, but *exactly facts*) is not sufficient for giving a well-grounded answer. Hence, one has to use the *hypothetic*-deductive method. Suppose that, according to *authentic* Brouwer’s intuitionism, in the propositional logic only the classical logic law of excluded middle and the classical logic law concerning double negation are not reliable. Under this condition (hypothesis 1), Brouwer ought to bequite satisfied by Heyting’s propositional logic calculus [6], [7]. Suppose that, according to *authentic* Brouwer’s intuitionism, *all* laws of the classical propositional logic are not reliable. Under this very strong (extremely challenging) condition (hypothesis 2), Brouwer ought to be*not**quite* (but only partly) satisfied by Heyting’s intuitionism-logic-formalization. There are some grounds for the guess that also Wittgenstein [8], [9] has been *not**quite* satisfied by Heyting’s formalization of intuitionism. The present paper is devoted to investigating the hypothesis 2 by representing (modeling) it asalogically formalized multimodal axiomatic theory Ф0 of universal epistemology combining a-priori-ism and empiricism consistently. The axiom schemes of the formal theory Ф0 are the following twelve ones. The logic symbols are used here in their *classical* logic meanings. In this paper, the symbols α, β, ω, belonging to meta-language, denote any formulae of the theory Ф0, while the symbol Ω, belonging to meta-language, denote any element of the set of “*perfection-*modalities”, which set is precisely defined in semantics of object-language of Ф0.

AX-0: (Aα ⊃ ©), where the symbol ©, belonging to meta-language, stands for *any* theorem of the *classical* propositional logic.

AX-1: Aα ⊃ (Ωα ⊃ α).

AX-2: Aα ⊃ (Ω(α ⊃ β) ⊃ (Ωα ⊃ Ωβ)).

AX-3: Aα ↔ (Kα & (¬◊¬α & ¬◊Sα & (β ↔ Ωβ))).

AX-4: Eα ↔ (Kα & (◊¬α ∨ ◊Sα ∨ ¬(β ↔ Ωβ))).

AX-5: Ωβ ⊃ ◊β.

AX-6: (β & Ωβ) ⊃ β.

AX-7: (ti=+=tk) ↔ (G[ti] ↔ G[tk]), where: the symbols ti and tk, belonging to meta-language, stand for any (arbitrarily taken) *terms* of the theory Ф0; the sign “=+=” stands for the relation of “*formal-axiological equivalence* of terms” of Ф0; and square-bracketing denotes assigning the *ontological value* to the square-bracketed term *in a definite standard interpretation* of the theory Ф0. (The meaning of square-bracketing is defined precisely in the semantics of Ф0.)

AX-8: (ti=+=g) ⊃ G[ti].

AX-9: (ti=+=b) ⊃ W[ti].

AX-10: (Gα ⊃ ¬Wα).

AX-11: (Wα ⊃ ¬Gα).

Definition DF-1: ◊ω is a *name* of/for ¬¬ω (where ω is a formula of Ф0).

Ω stands for any element of the set ℜ = {, K, T, F, P, D, C, Y, G, O, B, U, J}. Elements of ℜ are called “*perfection*-modalities”. The set of *perfection*-modalities ℜ is a subset of the set ℑ of the modalities combined by Ф0, namely, ℑ = {◊, , K, A, E, S, T, F, P, D, C, Y, G, W, O, B, U, J}.

Symbols ◊ and  stand for the modalities “possible” and “necessary”, respectively. Symbols K, A, E, S, T, F, P, D, respectively, stand for modalities “agent *Knows* that…”, “agent *A-priori knows* that…”, “agent *Empirically knows* that…”, “under some conditions in some space-and-time a person (immediately or by means of some tools) has *Sensual verification* that…”, “it is *True* that…”, “person has *Faith* that…”, “it is *Provable* that…”, “there is *an algorithm* (a machine could be constructed) *for Deciding* that…”.

Symbols C, Y, G, W, O, B, U, J, respectively, stand for modalities “it is *Consistent* that…”, “it is *Complete* that…”, “it is *Good* that…”, “it is *Wicked* that…”, “it is *Obligatory* that …”, “it is *Beautiful* that …”, “it is *Useful* that …”, “it *is Joyful,* *pleasant* that…”.

Precise definitions of “alphabet of the object-language of Ф0”, “term of Ф0”, and “formula of Ф0” can be provided. A definition of *semantics of object-language* of the formal theory Ф0 can also be provided. The formal theory Ф0 is a result of significant *generalizations* of the formal theories Θ [10] and Σ+C [11]. A proof of consistency of Ф0 is realized by constructing a *model* of/for Ф0, i.e. by providing an *interpretation* in which all the axioms are true and the rules of inference preserve truth.

With respect to R. Blanché’s idea [12] of psychologic-pedagogic and even heuristic value of graphic-modeling logic-structures of systems of abstract notions, the logical structure of the system of abstract notions united by the theory Ф0 may be modeled graphically by the below-presented logical square-and-hexagon of conceptual opposition of the meta-theoretic properties.

Let us extend the realm of epistemic-modality investigations from the set of propositions to the set of either-propositions-or-theories. Elements of the above-defined sets ℜ and ℑ are modalities *de dicto*, i.e. each of them is attached to *a dictum*, which is an *affirmation* of something. If not only propositions but also theories can be affirmed, one can consider theories as dictums and apply to them the modalities *de dicto*. Certainly, this is an extension of the ordinary usage of modal notion but the novelty is worth investigating.

Let variable “t” stand for a theory having a recursively enumerable set of axioms. Let “Emp(t)” stand for the meta-theoretic property “theory t (as a whole) is *empirical*”. “Apr(t)” stands for the meta-theoretic property “theory t is a system of *a-priori* knowledge. “Cla(t)” – “theory t is based on the *classical* logic”. “Con(t)” – “theory t is *consistent*”. “Com(t)” – “theory t is *complete*”. “Dec(t)” – “theory t is *decidable*”.

Notions “Apr(t)” and “Emp(t)” are defined as follows. DF-1: Apr(t) ↔ (Cla(t) & Con(t) & Com(t) & Dec(t)). DF-2: Emp(t) ↔ (⎤Cla(t) ∨ ⎤Con(t) ∨ ⎤Com(t) ∨ ⎤Dec(t)).

The system of logical interconnections among the meta-theoretic notions is modeled graphically by the below-located logical square-and-hexagon of conceptual opposition.

⎤Cla(t) ∨ ⎤Con(t) ∨ ⎤Com(t) ∨ ⎤Dec(t)

⎤(Cla(t) ∨ ⎤Com(t) ∨ ⎤Dec(t)

Cla(t) & ⎤Con(t) & Dec(t)

⎤Cla(t) ∨ Con(t) ∨ ⎤Dec(t)

Cla(t) & Com(t) & Dec(t)

(Cla(t) & Con(t) & Com(t) & Dec(t)

**Kp**

**Kp**

Fig.1. The meta-theoretic square and hexagon uniting *empirical* theories with *a-priori* ones

Fig.1 is an outcome of significant *explication* and *generalization* of the *meta-theoretic*square and hexagon published in [13]. In September 9-13, 2022, the novel (substantially explicated and generalized) *meta-theoretic*square and hexagon was submitted while the oral presentation at the 7th World Congress on the Square of Oppositionin KU Leuven [14] as a graphic model of logical relations among the relevant epistemic notions precisely defined (indirectly) by the logically formalized multimodal axiomatic system Σ+C formulated originally in [11]. But for the geometric modeling of logical relations among the epistemic concepts synthesized by Ф0, the hexagon (containing the square of opposition) presented by Fig.1 fits much better. Also it is worth noting here that according to [11], [14], [15], being formulated as an absolutely *universal* principle, Kant’s statement of *a-priori*-ness of logic (as a whole) and of mathematics (as a whole) is not quite adequate.

**References**

[1] Hilbert, D. (1990). *Foundations of Geometry [Grundlagen der Geometrie]*. Open Court Publishing Co., La Salle, IL.

[2] Hilbert, D. (1996). The logical foundations of mathematics. In Ewald, W. B. (Ed.). *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*. *Vol. 2*. Oxford University Press, Oxford, New York, pp. 1134–1147.

[3] Brouwer, L. E. J. (1975). On the unreliability of the logical principles. In A. Heyting (Ed.). *The collected works of L.E.J. Brouwer* (pp. 107-111). Amsterdam: North Holland.

[4] Brouwer, L. E. J. (1983). Intuitionism and formalism. In P. Benacerraf and H. Putnam (Eds.). *Philosophy of Mathematics* (pp. 77-89). Prentice Hall, Englewood Cliffs N.J., Cambridge: Cambridge University Press.

[5] Dalen, D. Van. (1981). *Brouwer’s Cambridge lectures on intuitionism*. Cambridge: The University Press.

[6] Heyting, A. (1956). *Intuitionism: an introduction*. Amsterdam: North-Holland.

[7] Heyting, A. (ed.). (1975). *L. E. J. Brouwer: Collected Works* (Volume 1: *Philosophy and Foundations of Mathematics*), Amsterdam and New York: Elsevier.

[8] Wittgenstein, L. (1975). *Philosophical remarks*. Oxford: Blackwell.

[9] Wittgenstein, L. (1978). *Remarks on the Foundations of Mathematics.* Oxford: Basil Blackwell.

[10] Lobovikov, V. O. (2019). Wittgenstein’s conception of unreliability of all laws of classical logic in case of unreliability of the law-of-the-excluded-middle: explicating this conception by means of a formal-axiomatic-epistemology-theory called “Theta”. In A. Siegetsleitner, A. Oberprantacher, M.-L. Frick (eds.) *Proceedings of the 42nd International Wittgenstein Symposium (Kirchberg am Wechsel, Austria, August 4–10, 2019) “Crisis and Critique: Philosophical Analysis and Current Events”* (pp. 145-147). Kirchberg am Wechsel, Austrian Ludwig Wittgenstein Society.

[11] Lobovikov, V. O. (2021). A Logically Formalized Axiomatic Epistemology System Σ+C and Philosophical Grounding Mathematics as a Self-Sufficing System. *Mathematics*, 9, 1859. <https://doi.org/10.3390/math9161859>

[12] Blanché, R. (1966). *Structures intellectuelles.* Paris, Vrin.

[13] Lobovikov, V. O. (2015). A meta-theoretical interpretation of the logical square and hexagon of opposition. In: Jean-Yves Beziau, Safak Ural, Arthur Buchsbaum, Iskender Tasdelen, Vedat Kamer (eds.). *Handbook of the 5th World Congress and School on Universal Logic (June 20- 30, 2015),* Istanbul, Turkey: University of Istanbul, pp. 346-348.

[14] Lobovikov, V.O. (2022). A new metatheoretic square and hexagon uniting empirical theories with a-priori ones, and uniting theories based on the classical logic with ones based on a nonclassical logic, in: Lorenz Demey, Dany Jaspers & Hans Smessaert (eds.). *Handbook of the 7th World Congress on the Square of Opposition “Square 2022”, September 9-13, 2022* ([www.square-of-opposition.org](http://www.square-of-opposition.org)). Leuven, Belgium, pp. 53-54.

[15] Lobovikov, V.O. (2023). Combining Universal Epistemology with Formal Axiology in a Multimodal Formal Axiomatic Theory “Sigma + 2C”, and Philosophical Foundations of Mathematics. *Respublica Literaria*. Vol. 4. no. 4. pp. 88-113.